Thursday July 2, 2020

Speaker: Lina Li, University of Illinois Urbana-Champaign

Title: On the number of Integer colorings with forbidden rainbow sums

Abstract:

In 1974, Erdős and Rothchild proposed a problem of determining the optimal n-vertex graphs, which maximize the number of two-edge-colorings without monochromatic triangles. Since then, this new class of coloured extremal problems has been extensively studied by many researchers on various discrete structures, such as graphs, hypergraphs, Boolean lattices and additive sets.

For a set of integers $A \subseteq [n]$, an r-coloring of A is called rainbow sum-free if it contains no rainbow Schur triple. In this talk, we investigate a rainbow variant of the Erdős-Rothchild problem in the context of sum-free sets, which asks for the subsets of $[n]$ with the maximum number of rainbow sum-free r-colorings. We show that for $r = 3$, the interval $[n]$ is optimal, while for $r \geq 8$, the interval $[\lfloor n/2 \rfloor, n]$ is optimal. We also prove a stability theorem for any $r \geq 4$. The proofs rely on the hypergraph container method, and some ad-hoc stability analysis.

This is joint work with Yangyang Cheng, Yifan Jing, Guanghui Wang, and Wenling Zhou.

Thursday June 25, 2020

Speaker: Dr. Novi Herawati Bong, University of Delaware

Title: Strong Dimension and Threshold Strong Dimension of Graphs.

Abstract:

Let G be a graph and W a set of vertices of G. Then W is said to resolve G if for every pair u, v of vertices of G there is a vertex $w \in W$ such that the distances of u and v to w are distinct, i.e. $d_G(u, w) \neq d_G(v, w)$, and W strongly resolves G if for every pair u, v of vertices of G there is a vertex $w \in W$ such that either a shortest u–w path contains v or a shortest v–w path contains u. A smallest resolving set in a graph is called a basis and its cardinality the metric dimension and a smallest strong resolving set is called strong basis and its cardinality the strong dimension. The metric dimension and strong dimension of a graph may decrease if edges are added to the graph. The smallest metric dimension (strong dimension) among all graphs having G as spanning subgraph is called threshold dimension (threshold strong dimension) of G and is denoted by $\tau(G)$

(respectively, $\tau_s(G)$). We show that the threshold dimension and threshold strong dimension of a graph are not always the same. We determine bounds on the threshold strong dimension of a graph, and determine some structural properties of graphs with strong threshold dimension 2.

Thursday June 18, 2020

Speaker: Dr. Ellen Veomett, Saint Mary's College of California
Title: The Mathematician and the Mapmaker: Using Mathematics to Combat Gerrymandering
Abstract:

Gerrymandering is generally understood to be the drawing of political districts in order to benefit one group and dilute the voting power of another group. In recent years, mathematicians have become deeply involved in the geometry of redistricting and the evaluation of districting maps. In this talk, we will discuss some techniques that have recently been used in court cases to argue the presence of partisan gerrymandering. We will discuss research on particular metrics intended to detect gerrymandering, as well as statistical techniques using Markov Chain Monte Carlo.

Thursday June 11, 2020

Speaker: Shannon Ogden, University of Vitorica
Title: Total Roman Domination Edge-Supercritical and Edge-Removal-Supercritical Graphs
Abstract:

A total Roman dominating function on a graph G is a function $f : V(G) \to \{0, 1, 2\}$ such that every vertex v with $f(v) = 0$ is adjacent to some vertex u with $f(u) = 2$, and the subgraph of G induced by the set of vertices w such that $f(w) > 0$ has no isolated vertices. The weight of f is $\sum_{v \in V(G)} f(v)$. The total Roman domination number $\gamma_{tR}(G)$ is the minimum weight of a total Roman dominating function on G. A graph G is γ_{tR}-edge-supercritical if $\gamma tR(G+e) = \gamma tR(G)2$ for every $e \in E(\bar{G}) \neq \emptyset$, and γtR-edge-stable if $\gamma tR(G+e) = \gamma_{tR}(G)$ for every $e \in E(G)$. For an edge $e \in E(G)$ incident with a degree 1 vertex, we define $\gamma_{tR}(G - e) = \infty$. A graph G is γ_{tR}-ER-critical if $\gamma tR(Ge) > \gamma_{tR}(G)$ for every $e \in E(G)$, γ_{tR}-ER-supercritical if $\gamma_{tR}(G - e) \geq \gamma_{tR}(G) + 2$ for every $e \in E(G)$, and γ_{tR}-ER-stable if $\gamma_{tR}(G - e) = \gamma_{tR}(G)$ for every $e \in E(G)$. We characterize γ_{tR}-ER-critical and γ_{tR}-ER-supercritical graphs. In addition, we investigate connected γ_{tR}-edge-supercritical graphs and exhibit infinite classes of such graphs. We present a connection between γ_{tR}-ER-supercritical and γ_{tR}-edge-stable graphs, and similarly between γ_{tR}-edge-supercritical and γ_{tR}-ER-stable graphs.

Thursday June 4, 2020

Speaker: Dr. Érika Roldán, Technische Universität München

Title: Two Geometric Problems in Extremal Topological Combinatorics

Abstract: We consider two optimization questions for regular polyforms (polyiamonds and polyominoes). What is the maximum number of holes that a polyform with n tiles can enclose? And what is the minimum number of tiles required to construct a polyform with h holes? We completely solved this problem for polyiamonds and polyominoes by constructing polyforms that reach the maximum number of holes for any given amount of tiles. We also characterize the topological and geometric properties of polyiamonds and polyominoes that have maximally many holes. These results give an upper bound for the expectation of the rank of the first homology group of random polyiamonds and random polyominoes (a lower bound is also known, and it is also linear with respect to the number of tiles).

Joint work with Matthew Kahle and Greg Malen.

Thursday May 28, 2020

Speaker: Carla Groenland, University of Oxford

Title: Reconstruction from small cards

Abstract:

The graph reconstruction conjecture states that each graph G on at least 3 vertices can be reconstructed from the multiset of all induced subgraphs on $n-1$ vertices. Although this conjecture is notoriously difficult, it is easy to reconstruct some information about the graph, e.g. the degree sequence of G and whether G is connected. Suppose we instead consider the multiset of all induced subgraphs on k vertices. Can we reconstruct information about the graph with $k < n-1$? How large does k need to be? We improve the best known bounds of reconstructing the degree sequence and connectivity from 'small cards' to almost tight bounds using an algebraic tool.

This is based on joint work with Tom Johnston, Alex Scott and Jane Tan

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Thursday May 21, 2020

Speaker: Dr. Sophie Spirkl, Princeton University

Title: New results about pure pairs in graphs

Abstract:

A pure pair in a graph consists of two subsets A, B of the vertex set such that between them, either all or no edges are present. Graph classes with linear-size pure pairs satisfy the Erdős-Hajnal conjecture, but unfortunately H-free graphs (the class of graphs that doesn't contain a fixed graph H as an induced subgraph) only have linear-size pure pairs if H is contained in a four-vertex path. This motivates the question, given H, how large a pure pair can we guarantee in H-free graphs? I'll present background and some recent results. Based on joint work with Maria Chudnovsky, Jacob Fox, Alex Scott, and Paul Seymour.

Thursday May 14, 2020

Speaker: Dr. Fiona Skerman, Uppsala Univeristy

Title: Quasirandom-forcing tournaments

Abstract:

A tournament H is quasirandom-forcing if the following holds for every sequence $(G_n)_n$ of tournaments of growing orders: if the density of H in G_n converges to the expected density of H in a random tournament, then $(G_n)_n$ is quasirandom. Every transitive tournament with at least 4 vertices is quasirandom-forcing, and Coregliano et al. [Electron. J. Combin. 26 (2019), P1.44] showed that there is also a non-transitive 5-vertex tournament with the property. We show that no additional tournament has this property. This extends the result of Bucic et al. [arXiv:1910.09936] that the non-transitive tournaments with seven or more vertices do not have this property.

This is joint work with Robert Hancock, Adam Kabela, Dan Král', Taísa Martins, Roberto Parente and Jan Volec.

Thursday May 7, 2020

Speaker: Dr. Ray Karpman, Otterbein University

Title: Combinatorics of Latin Tableaux

Abstract

Latin tableaux are a generalization of Latin squares. They were first introduced in 2002 by Chow, Fan, Goemans, and Vondrak. Chow and his collaborators hoped to use Latin tableaux as a tool to prove Rota's basis conjecture – a question in linear algebra which has been open since the early 1990's. Their approach was foiled, however, because Latin tableaux are surprisingly mysterious objects. Nearly twenty years later, many natural questions about Latin tableaux remain unexplored. In this talk, we extend the notion of isotopy, a permutation group action, from Latin squares to Latin tableaux. We define isotopy graphs for Latin tableaux, which encode the structure of orbits under the isotopy action, and investigate the relationship between the shape of a Latin tableau and the structure of its isotopy graph. We show, for example, that for any positive integer d, there is a Latin tableau whose isotopy graph is a d-dimensional cube.

Thursday April 30, 2020

Speaker: Dr. Gal Kronenberg, University of Oxford

Title: Extremal problems of long cycles in random graphs

Abstract:

We consider the random version of some classical extremal problems in the context of long cycles. This type of problems can also be seen as random analogues of the Turán number of long cycles, established by Woodall in 1972. For a graph G on n vertices and a graph H, denote by $\mathrm{ex}(G, H)$ the maximal number of edges in an H-free subgraph of G. We consider a random graph $G \sim G(n, p)$ where $p > C/n$, and determine the asymptotic value of $\mathrm{ex}(G, C_t)$, for every $A * \log(n) < t < (1-\varepsilon)n$. The behaviour of $\mathrm{ex}(G, C_t)$ can depend substantially on the parity of t. In particular, our results match the classical result of Woodall, and demonstrate the transference principle in the context of long cycles. Using similar techniques, we also prove a robustness-type result, showing the likely existence of cycles of prescribed lengths in a random subgraph of a graph with a nearly optimal density (a nearly "Woodall graph"). If time permits, we will discuss some connections to size-Ramsey numbers of long cycles. Based on joint works with Michael Krivelevich and Adva Mond.

Thursday April 23, 2020

Speaker: Dr. Natasha Morrison, University of Cambridge

Title: Invertibility of Random Symmetric Matrices

Abstract:

Abstract:A well-known conjecture states that a random symmetric n by n matrix with entries in $\{-1, 1\}$ is singular with probability of order $(n^2)/(2^n)$. In this talk I will describe some recent work where we prove that the probability of this event is at most $\exp(-cn^{1/2})$. This improves the previous best known bound of $\exp(-cn^{1/4} * (\log n)^{1/2}))$, which was obtained by Ferber and Jain. Our main theorem is an inverse Littlewood-Offord theorem in Z_p^n, which is inspired by the method of hypergraph containers. This is joint work with Marcelo Campos, Leticia Mattos and Rob Morris.

Thursday April 15, 2020

Speaker: Dr. Eva Czabarka, University of South Carolina

Title: The midrange crossing constant and some of its uses

Abstract:

The crossing number of a graph is the minimum number of crossings it can be drawn in a plane. Let $\kappa(n,m)$ be the minimum crossing number of graphs with n vertices and (at least) m edges. Erdős and Guy conjectured and Pach, Spencer and Tóth proved that for any $m = m(n)$ satisfying $n << m << n^2$ the quatity $\lim_{n\to\infty} \frac{\kappa(n,m)n^2}{m^2}$ exists and is positive. The k-planar crossing number of a graph is the minimum crossing number obtained when we partition the edges of the graph into k subgraphs and draw them in k planes. Using designs and a probabilistic algorithm, the guaranteed factor of improvement α_k between the k-planar and regular crossing number is $\frac{1}{k^2}(1+o(1))$, while if we restrict our attention to biplanar graphs, this constant is $\beta_k = \frac{1}{k^2}$ exactly. The lower bound proofs require the existence of a midrange crossing constant. Motivated by this, we show that the midrange crossing constant exists for all graph classes (including bipartite graphs) that satisfy certain mild conditions. The regular midrange crossing constant was shown to be is at most $\frac{8}{9\pi^2}$; we present a probabilistic construction that also shows this bound.

Thursday April 9, 2020

Speaker: Dr. Michelle Delcourt, Ryerson University

Title: Progress towards Nash-Williams' Conjecture on Triangle Decompositions

Abstract:

Partitioning the edges of a graph into edge disjoint triangles forms a triangle decomposition of the graph. A famous conjecture by Nash-Williams from 1970 asserts that any sufficiently large, triangle divisible graph on n vertices with minimum degree at least $0.75n$ admits a triangle decomposition. In the light of recent results, the fractional version of this problem is of central importance. A fractional triangle decomposition is an assignment of non-negative weights to each triangle in a graph such that the sum of the weights along each edge is precisely one.

We show that for any graph on n vertices with minimum degree at least $0.827327n$ admits a fractional triangle decomposition. Combined with results

of Barber, Kühn, Lo, and Osthus, this implies that for every sufficiently large triangle divisible graph on n vertices with minimum degree at least $0.82733n$ admits a triangle decomposition. This is a significant improvement over the previous asymptotic result of Dross showing the existence of fractional triangle decompositions of sufficiently large graphs with minimum degree more than $0.9n$. This is joint work with Luke Postle.

Thursday April 2, 2020

Speaker: Dr. Margaret Bayer, University of Kansas

Title: Counting flags in Eulerian posets: The *cd*-index

Abstract:
The face lattices of convex polytopes and intervals in the Bruhat order of Coxeter groups are examples of Eulerian posets. The flag vector of an Eulerian poset counts the sequences of elements of specified ranks. The linear relations that hold for the flag vectors of all Eulerian posets are known, and it turns out the dimension of the flag vectors for posets of fixed rank is a number in the Fibonacci sequence. Jonathan Fine discovered that the flag vectors could be efficiently coded in a vector called the *cd*-index. This talk discusses the history of the *cd*-index, with a focus on inequalities, connections with other combinatorial parameters and relations to algebraic structures.

Thursday March 26, 2020

Speaker: Dr. Ruth Luo, University of California San Diego

Title: Super-pancyclic hypergraphs and bipartite graphs

Abstract:
Let $G = G(m, n, r)$ be a bipartite graph with parts $A \cup B$, such that $|A| = m, |B| = n$, and $deg(a) \geq r$ for every $a \in A$. Jackson proved that if $m \leq r$ and $n \leq 2r2$, then G contains a cycle that covers every vertex in A. He also conjectured that if G is 2-connected, the result holds with the relaxed bound $n \leq 3r5$. We prove Jackson's conjecture and classify the extremal examples. Furthermore, we discuss implications for Berge cycles in r-uniform hypergraphs, and show necessary and sufficient conditions for the existence of Berge cycles using any fixed set of vertices. This is joint work with Alexandr

Kostochka, Misha Lavrov, and Dara Zirlin